An Optimal Q-Guidance Scheme for Satellite Launch Vehicles

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A closed-loop steering logic based on an optimal Q-guidance is developed here. The guidance system drives the satellite launch vehicle along a two- or three-dimensional trajectory for placing the payload into a specified circular orbit. The modified Q-guidance algorithm makes use of the optimal required velocity vector, which minimizes the total impulse needed for an equivalent two-impluse transfer from the present state to the final orbit. The required velocity vector is defined as velocity of the vehicle on the hypothetical transfer orbit immediately after the application of the first impulse. For this optimal transfer orbit, a simple and elegant expression for the Q-matrix is derived. A working principle for the guidance algorithm in terms of the major and minor cycles, and also for the generation of the steering command, is outlined.

Introduction

DURING the past 30 years, several inertial closed-loop guidance schemes have been developed and successfully employed in a number of launch vehicles. Broadly, these schemes can be classified into two categories, namely, path adaptive schemes and perturbation schemes.[1-3] In the case of path adaptive schemes, the steering command is generated from the solution of the simplified equations of motion using the instantaneous state, the desired terminal state, and the system parameters. On the other hand, perturbation guidance schemes assume that the launch trajectory is defined completely before the launch and that the reference nominal trajectory is available. The problem is to find the optimum steering logic that forces the vehicle to follow the nominal trajectory closely. The approach is simple and the accuracy achieved is high. Since the nominal optimal trajectory is computed on the ground, a sophisticated mathematical model could be employed. The Q-guidance is prominent among the perturbation guidance schemes. Battin[4] presents an inspiring historical account of the development of Q-guidance. The present paper attempts to develop an optimal Q-guidance scheme for a three-dimensional trajectory of a satellite launch vehicle.

The Q-guidance uses the required velocity concept (Ref. 5, pp. 123-125) to guide a launch vehicle to the specified terminal orbit. The required velocity vector ($\boldsymbol{v}_r$) is a hypothetical quantity specified with respect to a given position vector at the current instant and the terminal position vector,[4] such that the launch vehicle with the velocity $\boldsymbol{v}_r$ on its free flight will reach the prefixed terminal point. Here, no condition is imposed on the terminal velocity vector for obtaining the required velocity vector. At the time of thrust cutoff, the actual velocity of the vehicle should attain the corresponding required velocity. This is achieved using the term "velocity-to-be-gained" $\boldsymbol{v}_g(t)$, which is defined as the required velocity $\boldsymbol{v}_r(t)$ minus the vehicle velocity $\boldsymbol{v}(t)$, and driving this quantity ($\boldsymbol{v}_g$) to zero through logics like, say, cross-product steering.[6,7] It is impractical to compute $\boldsymbol{v}_g$ at every instant and then try to drive $\boldsymbol{v}_g$ to zero towards the thrust cutoff. Instead, Battin[4,5] has derived a functional relationship for computing $\boldsymbol{v}_g$ through a differential equation,

$$\dot{\boldsymbol{v}}_g = -(Q\boldsymbol{v}_g + \boldsymbol{a}_T) \tag{1}$$

where $Q = \partial \boldsymbol{v}_r / \partial \boldsymbol{r}$, $\dot{\boldsymbol{v}}_g = \mathrm{d}\boldsymbol{v}_g/\mathrm{d}t$, and $\boldsymbol{a}_T$ is the thrust vector. The Q-matrix is computed using the nominal parameters. From the user's point of view, the guidance algorithm works in two cycles, the guidance minor cycle during which Q is kept constant, and the guidance major cycle during which the entries in Q are updated. The guidance problem can now be stated as the problem of determining the steering/thrust program for optimally driving $\boldsymbol{v}_g$ to zero. Martin[6,7] has considered three steering logics: 1) align the thrust vector $\boldsymbol{a}_T$ with the $\boldsymbol{v}_g$ direction, 2) direct $\boldsymbol{a}_T$ to cause $\boldsymbol{v}_g$ to align with its derivative, and 3) combine these two logics. The pros and cons of these logics have been extensively discussed by Battin[4] and Martin.[7,8] The formulation of the Q-guidance and the methodology of obtaining the steering law is well documented by Ferner and Schmitt,[9] Sarture,[10] and McAllister et al.[11] The computational aspect of the guidance logic through the dual mode of computer operation (major and minor cycle operation) has been considered by Gunkel.[12] Draper et al.[13] have derived the Q(VG: velocity-to-be-gained) differential equation and subsequently deduced the steering angles based on $\boldsymbol{v}_g \times \dot{\boldsymbol{v}}_g = 0$, and $\boldsymbol{v}_g \times \boldsymbol{a}_T = 0$. The physical implications of these two schemes are discussed by Battin[4] and Draper et al.[13] Using the linear optimal control theory, Sokkappa[14] has considered the fuel optimal guidance based on VG differential equation. Several variations of Q-guidance have been developed in the past.

In general, the Q-guidance depends strongly on the functional relationship governing the required velocity $\boldsymbol{v}_r$. Such a relationship has to be established with minimal assumptions for simplification. Owing to the use of the perturbation theory in the development of guidance system, one can afford to use a complex mathematical model for getting $\boldsymbol{v}_r$. Once such a functional relationship is defined, the determination of the Q-matrix using the nominal system state and parameters is straightforward. It is for this reason that $\boldsymbol{v}_r$ needs to be defined with great care. It is possible to interpret the required velocity in the following manner without altering the rationale stated earlier. For the vehicle moving along the powered flight path with the velocity $\boldsymbol{v}(t)$ at the current position $\boldsymbol{r}(t)$, if the thrust were to be cut off, the vehicle would follow an elliptical trajectory. Battin[4] has considered a single-impulse coasting trajectory for the computation of the required velocity. However, it is well known[15] that it is impossible to find a single-impulse transfer trajectory from a low Earth orbit to a higher Earth orbit (in general, the transfer between orbits of different size needs two or more impulses). The problem is further complicated if the planes of the initial and final orbits do not coincide. In the present study, an optimal two-impulse transfer is considered on the lines of Refs. 16 and 17 to determine the required velocity. At each instant of time, the guidance scheme specifies the point of entry (injection) into the desired orbit, the expression for the optimum required velocity,

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and the optimum Q-matrix. Throughout the analysis, it is assumed that the perturbative forces are small, and that the point of entry does not shift significantly from the nominal injection point. To simplify the analysis considerably without sacrificing the optimality, the apogee of the transfer orbit is assumed to lie on the terminal orbit. Its location, however, is not specified. It may be noted that the location of the first impulse is established by the current position vector $\boldsymbol{r}(t)$. The required velocity $\boldsymbol{v}_r$ is defined as the velocity of the vehicle on the transfer trajectory at $\boldsymbol{r}(t)$. The second impulse is applied at the apogee of the transfer orbit to circularize the orbit. The purpose of this paper is to find an optimal $\boldsymbol{v}_r$ for which the total effort for the two impulses is minimal. To the best knowledge of the authors, such an approach has not been reported in the open literature.

The velocity to be gained, $\boldsymbol{v}_g$, is defined as

$$\boldsymbol{v}_g(t) = \boldsymbol{v}_r(t) - \boldsymbol{v}(t) \tag{2}$$

Instead of determining $\boldsymbol{v}_g(t)$ at all instants, VG differential equation[4] can be used. The expression for the $\boldsymbol{Q}$-matrix in Eq. (1) is determined from the optimal $\boldsymbol{v}_r$. Then, using cross-product steering, the closed-loop guidance policy is derived.

Optimal Required Velocity Computation

Figure 1 shows the geometry of the initial elliptical trajectory i_o after the hypothetical thrust cutoff at $r(t)$, the coasting trajectory c, and the desired circular orbit d. After the application of the impulse $\Delta\boldsymbol{v}_1$ at I_1, the vehicle attains the velocity $\boldsymbol{v}_c$ at I_1 and subsequently traverses trajectory c. By applying another impulse $\Delta\boldsymbol{v}_2$ at I_2, the circular orbit velocity $(\mu/r_d)^{1/2}$ is obtained. The location of I_2 is not specified initially. The present paper determines the coasting transfer orbit c for which a measure of the total impulse requirement is minimal. For the optimal set of $\Delta\boldsymbol{v}_1$ and $\Delta\boldsymbol{v}_2$, the optimal entry point I_2 and the optimal vehicle velocity $\boldsymbol{v}_c^*$ at I_1 on orbit c get fixed. The required velocity $\boldsymbol{v}_r$ is now defined as the optimal $\boldsymbol{v}_c^*$. Note that the definition of required velocity is different from the classical one. Therefore, the resulting Q-matrix also differs from the standard Q-matrix.[4,5]

The orbital elements are taken to be the state variables in the problem of determining the optimal transfer orbit due to the following advantages:[17,18]

1) Along the ballistic arcs (thrust acceleration is zero) $\boldsymbol{x}$, the vector of five orbital elements, is constant; the sixth element, corresponding to time or equivalent parameter, can be found analytically (though it is not used in this analysis).

2) On the arcs, the orbital elements vary much less than the position vector $\boldsymbol{r}$ and velocity vector $\boldsymbol{v}$.

3) It is easy to handle the end conditions. Generally, it is sufficient to define the initial and final orbits i_o, d, with the locations of the impulses I_1 and I_2 left free. In the present context, however, the location I_1 is known.

In Fig. 2, ON is the line of intersection of the planes of the initial and desired orbits. Since, orbit d is circular, ON is treated as a reference axis. The orbit d crosses the equatorial plane at E_d. Let β_1 be the true anomaly[19] of the line ON measured from the perigee of the initial orbit. Let β_2 and ϕ_d, respectively, be the polar angles of OE_d with reference to ON, and OI_2 with reference to OE_d. The elliptical coasting arc c, with its apogee A_c at I_2, connects I_1 and I_2. In Fig. 3, δ is the transfer angle measured in the plane of the transfer orbit. Here $\delta = \pi - \delta_1$, where δ_1 is the true anomaly of I_1 in the transfer orbit plane. Let p_c and e_c be, respectively, the semilatus rectum and eccentricity of the coasting orbit c. Then

$$p_c = r(1 - e_c \cos\delta) = r_d(1 - e_c) \tag{3}$$

which gives, on simplification,

$$p_c = r r_d(1 - \cos\delta)/(r_d - r\cos\delta) \tag{4}$$

$$e_c = (r_d - r)/(r_d - r\cos\delta) \tag{5}$$

These parameters, not specified at the moment, are subject to optimization.

Geometric Relations Between Orbital Parameters

The geometric relations involving some of the orbital parameters are obtained with the help of spherical trigonometry.[20] From the spherical triangle $I_1 I_2 N$ subtended at O (Fig. 2), the following relations can be deduced:

$$\cos\delta = \cos(\beta_1 - \theta)\cos(\beta_2 + \phi_d) - \sin(\beta_1 - \theta) \times \sin(\beta_2 + \phi_d)\cos\phi \tag{6}$$

$$\cos\phi = \cos\gamma_1 \cos\gamma_2 - \sin\gamma_1 \sin\gamma_2 \cos\delta \tag{7}$$

$$\sin\gamma_1 = \sin(\beta_2 + \phi_d)\sin\phi/\sin\delta \tag{8}$$

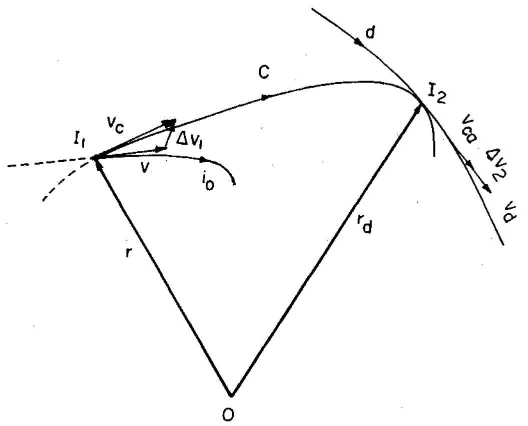

Fig. 1 Geometry of initial, transfer, and final desired orbits.

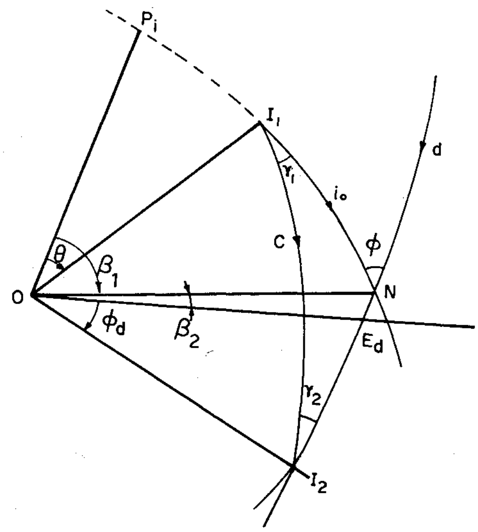

Fig. 2 Geometry of orbits projected on unit sphere.

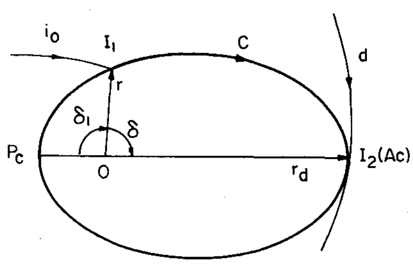

Fig. 3 Coasting transfer ellipse.

$$\sin \gamma_2 = \sin(\beta_1 - \theta) \sin \phi / \sin \delta \tag{9}$$

$$\cos \gamma_1 = \cos \gamma_2 \cos \phi + \sin \gamma_2 \cos(\beta_2 + \phi_d) \sin \phi \tag{10}$$

$$\cos \gamma_2 = \cos \gamma_1 \cos \phi + \sin \gamma_1 \cos(\beta_1 - \theta) \sin \phi \tag{11}$$

Here γ_1 and γ_2 represent rotations of the orbital plane from the initial to the transfer orbit and from the transfer to the desired circular orbit, respectively. These rotations are in the same directions. If γ_1 and γ_2 are in opposite directions, then a negative sign to γ_2 has to be introduced in the above relations. Using the trigonometric relations for the spherical triangle ENE_d (Fig. 4), one gets

$$\cos \phi = \cos i_d \cos i + \sin i_d \sin i \cos(\Omega - \Omega_d)$$

$$\sin \beta_2 = \sin \phi \sin i / \sin(\Omega - \Omega_d)$$

$$\sin(\beta_1 + \omega) = \sin \phi \sin i_d / \sin(\Omega - \Omega_d) \tag{12}$$

The variables ϕ_d, δ, γ_1, γ_2, p_c, and e_c differ from one transfer orbit to another. The rest of the variables are obtainable from the orbital parameters of the predetermined initial and desired orbit. The angle ϕ_d, which specifies the point I_2 (Fig. 2), can be taken as the independent variable. The other variables, p_c, e_c, δ, γ_1, and γ_2, can be expressed as functions of the variable ϕ_d and other known parameters with the help of Eqs. (4-8), respectively.

Determination of the Total Impulse Requirement

The optimal coasting transfer orbit is defined as the one for which the total impulse requirement (of the two impulses at I_1, and I_2) is minimal. The local horizontal (or transverse) component u and the local vertical (or radial) component v of the velocity vector for a given location on the elliptical trajectory are

$$u = h/r = (\mu p)^{1/2}/r, \qquad v = \mu e \sin \theta / h = (\mu/p)^{1/2} e \sin \theta$$

Let (u, v), and (u_c, v_c) be the components of the velocity vectors on the initial and coasting paths, respectively. The angle between u and u_c is equal to γ_1, i.e., the angle between the planes of the orbit. Then the square of the velocity increment needed at I_1 is

$$\begin{aligned}\Delta v_1^2 &= (v_c - v)^2 + (u_c^2 + u^2 - 2u_c u \cos \gamma_1) \\ &= \mu(e_c \sin \delta / p_c^{1/2} - e \sin \theta / p^{1/2})^2 \\ &\quad + \mu(p_c + p - 2(p p_c)^{1/2} \cos \gamma_1)/r^2\end{aligned} \tag{13}$$

The vertical components of the velocity vector for the coasting (apogee lies at I_2) and the circular orbit at I_2 are zero. The horizontal components at I_2 for the coasting trajectory and the desired orbit are, respectively,

$$u_{ca} = (\mu p_c)^{1/2}/r_d, \quad u_d = (\mu/r_d)^{1/2}$$

Hence

$$\begin{aligned}\Delta v_2^2 &= (u_d^2 + u_{ca}^2 - 2u_d u_{ca} \cos \gamma_2) \\ &= \mu(p_c/r_d + 1 - 2(p_c/r_d)^{1/2} \cos \gamma_2)/r_d\end{aligned} \tag{14}$$

The sum of Eqs. (13) and (14) is a measure of the total impulse requirement for transfer from the initial to the desired orbit. The location I_2 and the total impulse requirement depend on the selection of the coasting trajectory.

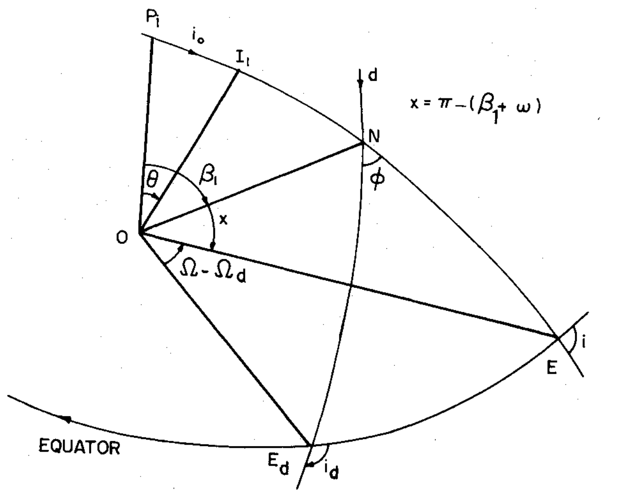

Fig. 4 Initial and final orbit in equatorial plane.

Minimization of the Impulse Requirement

For the optimization of the coasting transfer trajectory, let the cost function J be

$$\begin{aligned}J &= \Delta v_1^2 + \Delta v_2^2 \\ &= \mu[(e_c \sin \delta / p_c^{1/2} - e \sin \theta / p^{1/2})^2 + (p_c + p - 2(p p_c)^{1/2} \cos \gamma_1)/r^2 \\ &\quad + (p_c/r_d + 1 - 2(p_c/r_d)^{1/2} \cos \gamma_2)/r_d]\end{aligned} \tag{15}$$

This form of cost functional J is selected, first, because it is simpler than the expression for the square of the total impulse given by $(|\Delta v_1| + |\Delta v_2|)^2$. Second, it is a convex functional and is preferred over the standard cost functional of the form $(|\Delta v_1| + |\Delta v_2|)$. As points I_1 and I_2 lie on the elliptical trajectory, the principle of conservation of energy yields

$$v_c^2/2 - \mu/r = v_{ca}^2/2 - \mu/r_d \tag{16}$$

where $v_c^2 = \mu(e_c^2 \sin^2 \delta / p_c + p_c/r^2)$, $v_{ca}^2 = \mu p_c / r_d^2$.

Similarly

$$v_i^2 = \mu(e^2 \sin^2 \theta / p + p/r^2) \tag{17}$$

Using these identities in Eq. (23), and simplifying, yields

$$\begin{aligned}J &= \mu[2p_c/r_d^2 - 2ee_c \sin \delta \sin \theta / (p p_c)^{1/2} - 2(p p_c)^{1/2} \cos \gamma_1 / r^2 \\ &\quad - 2(p_c/r_d^3)^{1/2} \cos \gamma_2 + v_i^2 + 2(r_d - r)/r r_d + 1/r_d]\end{aligned} \tag{18}$$

The necessary condition for optimality for this unconstrained problem can be expressed as

$$\partial J / \partial \phi_d = 0 \tag{19}$$

as ϕ_d is the only variable under control. On simplification (see Appendix for details), the optimality condition can be written as

$$\begin{aligned}&[2(1 - x_8) x_8^{1/2} x_3 / x_6 x_5^{3/2} + e r_d (1 - x_8)^2 x_2 x_3 \sin \theta / (1 - x_1)^2 x_5 \\ &\quad - (r_d - r) x_3 x_4 p^{1/2} / (1 - x_1) x_2 x_5 - (r_d - r) x_3^2 / r_d^{3/2} (1 - x_1) x_5 x_2 \\ &\quad + 2e(r_d - r) x_2 x_3 \sin \theta / p^{1/2} (1 - x_1) x_5 - 2e(r_d - r) x_1 x_3 \sin \theta \\ &\quad \div r_d r (1 - x_1) x_2 p^{1/2} + 2p^{1/2} \sin(\beta_2 + \phi_d) \sin(\beta_1 - \theta) \sin^2 \phi \\ &\quad \div r^2 x_2^3 - 2 \sin^2(\beta_1 - \theta) \sin^2 \phi / r_d^{3/2} x_1 x_2] = 0\end{aligned} \tag{20}$$

It can be seen that Eq. (20) is highly nonlinear and that it is practically impossible to find an analytical closed-form optimal solution for ϕ_d. Therefore, the solution to ϕ_d has to be found by iteration. For any arbitrary starting value for ϕ_d, convergence to the true value cannot be guaranteed. It may be noted at this juncture that the perturbation guidance assumes the availability of the state and parameters along the nominal trajectory. Also, it is assumed that the deviation of the actual trajectory from the nominal is small. It is therefore possible to determine a nominal ϕ_d which, in turn, could be used as a good starting value for the iterations.

Equation (20) can be represented symbolically by the relation $F(\phi_d) = 0$. Then, using Taylor's series expansion, one could write

$$\phi_d = \phi_{d0} - F(\phi_{d0})/G(\phi_{d0}), \quad G(\phi_{d0}) = \partial F(\phi_d)/\partial\phi_d|_{\phi_d=\phi_{d0}} \tag{21}$$

where ϕ_{d0} is the initial guess for ϕ_d.

Differentiating x_i, $i = 1, \ldots, 7$ with respect to ϕ_d, Eq. (A17), one gets

$$x_1' = x_3, \qquad x_2' = x_1 x_3/x_2, \qquad x_3' = x_1,$$

$$x_4' = (\cos\phi - x_3 x_4)/x_1, \qquad x_5' = r x_3, \qquad x_6' = x_3/2x_6$$

where prime (′) indicates the partial derivative of the respective variable with respect to ϕ_d. Using these, the expression for $G(\phi_d)$ can be written as

$$\begin{aligned} G(\phi_d) = {} & [(2(1-x_8)x_3 x_8^{1/2}/x_6 x_5^{3/2})(x_1/x_3 - x_3/2(1-x_8) - 3r x_3/2x_5) \\ & + (e x_2 x_3^2 \sin\theta/(1-x_1)x_5)(r_d(1-x_8)^2/(1-x_1) + 2(r_d - r)/p^{1/2}) \\ & \times (x_1(x_2^2 + x_3^2)/x_1^2 x_3^2 - 2/(1-x_1) - r/x_5) - ((r_d - r)x_3/(1-x_1) \\ & \times x_2 x_5)(p^{1/2}((\cos\phi - x_3 x_4)/x_1 + x_1 x_4/x_3) + 2x_1/r_d^{3/2}) - (x_4 p^{1/2} \\ & + x_3/r_d^{3/2})((1/(1-x_1) + x_1/x_2^2 + r/x_5)x_3) - 2e(r_d - r)((x_1^2 - x_3^2) \\ & - x_1 x_3^2(1/(1-x_1) + x_1/x_2^2) \sin\theta/(r r_d(1-x_1)x_2 p^{1/2})) \\ & + 2p^{1/2}\sin(\beta_1 - \theta)\sin^2\phi\,(\cos(\beta_2 + \phi_d) - 3\sin(\beta_2 + \phi_d) \\ & \times x_1 x_3/x_2^2)/r^2 x_2^3 + 2\sin^2(\beta_1 - \theta)\sin^2\phi\,\tan 2\delta\, x_3/2x_2] \end{aligned} \tag{22}$$

Equation (21) is solved iteratively for ϕ_d until the differences between successive estimates become small. Each iterative step of the solution algorithm is as follows:

1) At the given instant, read the nominal value of ϕ_d, say, ϕ_{dn}.
2) Set $\phi_{d0} = \phi_{dn}$.
3) Calculate $\phi_{d1} = \phi_{d0} - F(\phi_{d0})/G(\phi_{d0})$, $\phi_{d2} = \phi_{d1} - F(\phi_{d1})/G(\phi_{d1})$.
4) Calculate $d_1 = \phi_{d1} - \phi_{d0}$, $d_2 = \phi_{d2} - \phi_{d1}$.
5) Calculate $\phi_d = \phi_{d2} - d_1/(d_2 - 1)$.
6) Set $\phi_{d0} = \phi_d$, and go to step 3 if there is no convergence.

Since, the initial guess is closer to the actual value of ϕ_d, it is expected that the solution to the optimization of a convex functional J will converge quadratically to the optimum value (ϕ_d^*).

Modified Q-Guidance Algorithm

The mechanization of the Q-guidance algorithm requires evaluation of the Q-matrix and derivation of the steering logic. The details of the derivation of Q-matrix are given in the Appendix. The philosophy of the guidance policy adopted here is to fix the injection point at each instant and then optimally steer the vehicle from the perturbed state to the new injection point. The analysis is based on the perturbation theory. The Q-matrix written below in the present case is much simpler than the one derived by Battin[4,5] for a single-impulse transfer to reach a fixed target in a given time. For that case, Battin[4] states that the derivation runs into fourteen pages. The Q-matrix derived in the Appendix can be written as

$$\boldsymbol{Q} = \mu/r^3(r_1 v_{r1} + r_2 v_{r2} + r_3 v_{r3}) \begin{bmatrix} r_1^2 & r_1 r_2 & r_1 r_3 \\ r_2 r_1 & r_2^2 & r_2 r_3 \\ r_3 r_1 & r_3 r_2 & r_3^2 \end{bmatrix} \tag{23}$$

As expected, the symmetry of the Q-matrix[4] is obvious from Eq. (23). The components of the vector $\boldsymbol{r}$ in Eq. (23) come directly from the inertial measurement unit (IMU), while the components of $\boldsymbol{v}_r$ are computed from the optimal ϕ_d obtained earlier, Eq. (21). The entries in Q are updated at each guidance major cycle.

Steering

The problem of steering the launch vehicle to the terminal condition is equivalent to driving the velocity-to-be-gained (VG) vector to zero. The $\boldsymbol{v}_g(t)$ is computed using the relation:

$$\dot{\boldsymbol{v}}_g(t) = -\boldsymbol{Q}\boldsymbol{v}_g - \boldsymbol{a}_T \tag{24}$$

As mentioned in the beginning, several steering logics have been suggested in the literature.[4,7,10] The cross-product steering[4] of the following form is considered first.

$$\boldsymbol{v}_g \times \dot{\boldsymbol{v}}_g = 0 \tag{25}$$

Using Eqs. (24) and (25), the thrust vector $\boldsymbol{a}_T$ can be expressed as

$$\boldsymbol{a}_T = (\alpha\boldsymbol{v}_g + \boldsymbol{v}_g \times \boldsymbol{y})/(\boldsymbol{v}_g \cdot \boldsymbol{v}_g) \tag{26}$$

where $\boldsymbol{y} = \boldsymbol{v}_g \times \boldsymbol{Q}\boldsymbol{v}_g$ and $\alpha = [(\boldsymbol{a}_T \cdot \boldsymbol{a}_T)(\boldsymbol{v}_g \cdot \boldsymbol{v}_g) - \boldsymbol{y} \cdot \boldsymbol{y}]^{1/2}$.

The cross-product steering, Eq. (26), gives the direction of the thrust vector for the predetermined $(a_T^2 = \boldsymbol{a}_T \cdot \boldsymbol{a}_T)$, $\boldsymbol{Q}$, and $\boldsymbol{v}_g$. It may be noted that the expression for the thrust program (26) is simpler than that reported by Martin.[7] Battin[4] advocates cross-product steering of the following form:

$$\boldsymbol{v}_g \times (\gamma\boldsymbol{q} - \boldsymbol{a}_T) = 0 \tag{27}$$

where γ = scalar mixing parameter

$$\boldsymbol{q} = \dot{\boldsymbol{v}}_g + \boldsymbol{a}_T$$

(since $\dot{\boldsymbol{v}}_g = \dot{\boldsymbol{v}}_r - \dot{\boldsymbol{v}} = \dot{\boldsymbol{v}}_r - \boldsymbol{g} - \boldsymbol{a}_T = \boldsymbol{q} - \boldsymbol{a}_T$)

Equation (27), together with Eq. (24), on simplification, reduces to

$$\boldsymbol{a}_T \times \dot{\boldsymbol{v}}_g = \gamma\, \boldsymbol{v}_g \times \boldsymbol{Q}\boldsymbol{v}_g \tag{28}$$

After defining $\boldsymbol{z} = \gamma\, \boldsymbol{v}_g \times \boldsymbol{Q}\boldsymbol{v}_g$ and $\beta = [(\boldsymbol{a}_T \cdot \boldsymbol{a}_T)(\boldsymbol{v}_g \cdot \boldsymbol{v}_g) - \boldsymbol{z} \cdot \boldsymbol{z}]^{1/2}$, the thrust control vector can be written as

$$\boldsymbol{a}_T = (\beta\boldsymbol{v}_g + \boldsymbol{v}_g \times \boldsymbol{z})/(\dot{\boldsymbol{v}}_g \cdot \boldsymbol{v}_g) \tag{29}$$

In practice, the scalar mixing parameter is determined after a number of ground-based simulations. The simplest steering algorithm comes from

$$\boldsymbol{v}_g \times \boldsymbol{a}_T = 0$$

that is,

$$\boldsymbol{a}_T = (\boldsymbol{a}_T \cdot \boldsymbol{a}_T)^{1/2}\,\hat{\boldsymbol{v}}_g, \qquad \hat{\boldsymbol{v}}_g = \text{unit}\,(\boldsymbol{v}_g) \tag{30}$$

It is observed from the literature survey[3] that the law of the form (26) is more popular with the analysts.

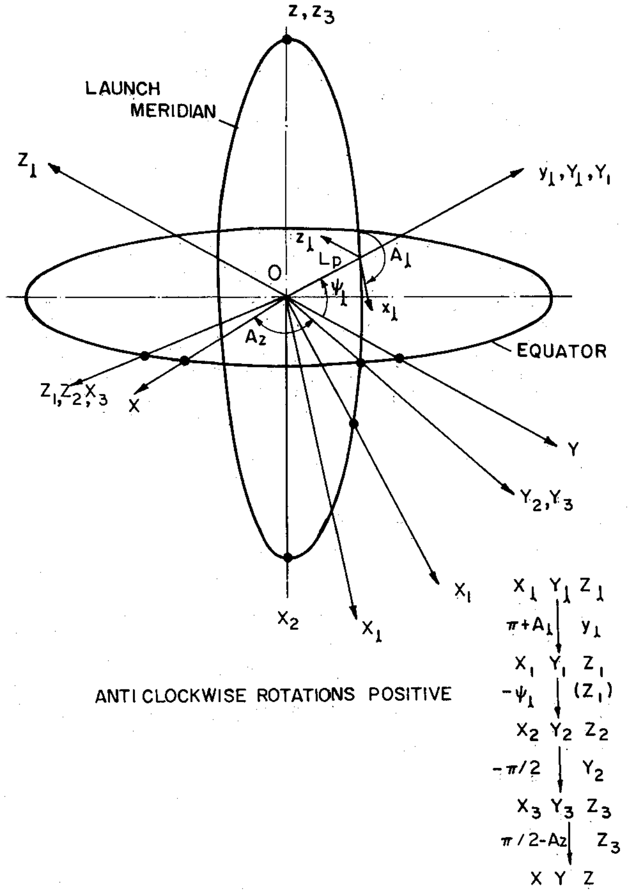

Fig. 5 Transformation between geocentric and INS-inertial reference frames.

Coordinate Frame

The selection of proper coordinates plays a dominant role in the successful implementation of a guidance algorithm. The two right-handed inertial reference frames used in most launch vehicle guidance systems are: 1) the Earth-centered equatorial inertial reference frame with one axis passing through the Vernal Equinox, the second passing through the North Pole, and the third completing the system; 2) the launch point fixed INS frame of reference, with one axis coming out of the launch point along the plumb line, and the second at a fixed orientation (aiming azimuth) with respect to the north-south line on the local horizontal plane, and the third, again, completing the axes system. All the trajectory parameters are usually defined in the inertial, Earth-centred coordinate frame. On the other hand, the current position and velocity vectors are supplied in the INS frame. It is, in general, possible to define a moving guidance frame fixed to the launch vehicle or the injection point. The disadvantage of these frames is the necessity to redefine the guidance frame as one marches along the trajectory. To overcome this problem, the Earth-centered inertial frame itself is selected as the guidance frame. The matrix $\boldsymbol{L}$ defined below transforms the vector from the INS frame (X_ℓ, Y_ℓ, Z_ℓ) to the inertial frame (X, Y, Z) illustrated in Fig. 5. It can be noted that the transpose of the matrix $\boldsymbol{L}$ defines the inverse transformation.

$$
L=\begin{bmatrix} -\cos A_z \sin\psi_\ell \cos A_\ell - \sin A_z \sin A_\ell & \cos A_z \cos\psi_\ell & \cos A_z \sin\psi_\ell \sin A_\ell - \sin A_z \cos A_\ell \\ -\sin A_z \sin\psi_\ell \cos A_\ell + \cos A_z \sin A_\ell & \sin A_z \cos\psi_\ell & \sin A_z \sin\psi_\ell \sin A_\ell + \cos A_z \cos A_\ell \\ \cos\psi_\ell \cos A_\ell & \sin\psi_\ell & -\cos\psi_\ell \sin A_\ell \end{bmatrix} \tag{31}
$$

where (A_z, ψ_ℓ) = argument of the launch meridian and latitude of the launch point, and A_ℓ = azimuth of the launch plane at launch point (positive clockwise from the north). The elements of the matrix L have to be computed only once or can be read as initial data since the entries in L are constant for a specified launch station and launch time.

The required velocity vector $\boldsymbol{v}_r$ is the velocity vector at I_1 on the optimal transfer orbit. It consists of two components, the vertical component v_{rc} (= optimal v_c^*) along the radius vector $\boldsymbol{r}$, and the horizontal component u_{rc} (= optimal u_r^*) along the transverse direction. The vector $\boldsymbol{v}_r$ lies in the transfer orbit plane. For the implementation of the guidance algorithm, it is necessary to express $\boldsymbol{v}_r$ in the XYZ frame/guidance frame $[\boldsymbol{v}_r = (v_{r1}, v_{r2}, v_{r3})]$. To accomplish this, two successive transformations are defined, such that (v_{r1}, v_{r2}, v_{r3}) can be found in terms of u_{rc} and v_{rc}. The first transformation defines the plane change from the transfer orbit plane to the initial elliptic orbit plane, so that $\boldsymbol{v}_r$ is defined along the vertical, transverse, and orbit normal directions on the initial orbital plane. The second transformation converts the vector from the above orbital frame of reference to the XYZ frame. The total transformation is given by the relation:

$$
\begin{bmatrix} v_{r1} \\ v_{r2} \\ v_{r3} \end{bmatrix} = \boldsymbol{V} \begin{bmatrix} v_{cr} \\ u_{cr}\cos\gamma_1^* \\ -u_{cr}\sin\gamma_1^* \end{bmatrix} \tag{32}
$$

where

$$V_{11} = \cos\Omega\cos(\omega+\theta) - \sin\Omega\sin(\omega+\theta)\cos i$$

$$V_{12} = -\cos\Omega\sin(\omega+\theta) - \sin\Omega\cos(\omega+\theta)\cos i$$

$$V_{13} = \sin\Omega\sin i$$

$$V_{21} = \sin\Omega\cos(\omega+\theta) + \cos\Omega\sin(\omega+\theta)\cos i$$

$$V_{22} = -\sin\Omega\sin(\omega+\theta) + \cos\Omega\cos(\omega+\theta)\cos i$$

$$V_{23} = -\cos\Omega\sin i, \quad V_{31} = \sin(\omega+\theta)\sin i$$

$$V_{32} = \cos(\omega+\theta)\sin i, \quad V_{33} = \cos i$$

γ_1^* is the optimal plane change from i_o to c, and $(\Omega, \omega, i, \theta)$ are the orbital parameters, Eqs. (A7-A8), for the orbit i_o. The matrix V has to be computed at every guidance major cycle.

Implementation of the Guidance Algorithm

Sequence of events in the implementation of the guidance algorithm are:

1) Specification of the input system parameters. The following parameters are stored on the onboard computer:
   a) History of the thrust magnitude a_T.
   b) A_ℓ = aiming azimuth (positive clockwise from the north, Fig. 6).
   c) A_z = argument of the launch meridian (Fig. 6).
   d) ψ_ℓ = geographic latitude of the launch point (Fig. 6).
   e) Ω_d = argument of the ascending node for the desired circular orbit.
   f) i_d = inclination of the desired circular orbit.
   g) r_d = radius of the desired circular orbit.
   h) Guidance major and minor cycle times (typically 2-5 and 0.1 s, respectively).
2) Initialization of the guidance algorithm. To initialize the algorithm the following steps are taken:
   a) Computation of the elements of the matrix $\boldsymbol{L}$.

b) Computation of the initial value of the matrix $\boldsymbol{V}$.
c) Transformation of the initial $\boldsymbol{r}_0$, $\boldsymbol{v}_0$ in the INS frame to the XYZ frame.
d) Determination of the initial value of ϕ_d from the nominal injection point.
e) Determination of $\boldsymbol{v}_{r0}$ and $\boldsymbol{Q}_0$ with the nominal ϕ_d.
f) Computation of the initial value for $\boldsymbol{v}_g$ for use at the very first minor cycle ($\boldsymbol{v}_{g0} = \boldsymbol{v}_{r0} - \boldsymbol{v}_0$).

3) Guidance major cycle computation. The steps involved in the major cycle are:
   a) Transformation of the position vector $\boldsymbol{r}$ and velocity vector $\boldsymbol{v}$ received from the INS into the inertial XYZ frame.
   b) Computation of the orbital parameters of the initial elliptical orbit, Eqs. (A2) and (A8).
   c) Determination of β_1 and β_2, Eq. (12).
   d) Using Eq. (21), determination of the near-optimal ϕ_d^* iteratively for which the initial guess ϕ_{d0} is equal to ϕ_d of the previous guidance major cycle.
   e) Determination of δ^*, e_c^*, p_c^* and γ_1^* using ϕ_d^* from Eqs. (6), (5), (3), and (8), respectively.
   f) Computation of $u_{cr} = u_c^*$ and $v_{cr} = v_c^*$, the horizontal and vertical components of the required velocity vector.
   g) Evaluation of the components of $\boldsymbol{v}_r$ in the XYZ frame, Eq. (32).
   h) Computation of the Q-matrix elements, Eq. (23).
   i) Initialization of $\boldsymbol{v}_g$ for use in the minor cycle.

4) Guidance minor cycle computation. For the minor cycle, employing one of the steering logics Eqs. (26), (29) or (30), one has to:
   a) Compute the components of $\boldsymbol{v}_g$ from VG differential equation at each cycle, Eq. (24).
   b) Determine the component of the thrust vector from the appropriate steering logic.
   c) Find the pitch and yaw thrust attitude angles (α_p, α_y) using the following relations: $\cos\alpha_p = (a_{Tx}^2 + a_{Ty}^2)^{1/2}/a_T$ and $\cos\alpha_y = a_{Ty}/(a_{Tx}^2 + a_{Ty}^2)$.
   d) Generate the thrust cutoff command when the magnitude of $\boldsymbol{v}_g$ becomes smaller than a prespecified value.

Note that the terminal guidance can be employed for a brief period before nominal thrust cutoff. During this phase, a single-impulse approximation can be utilized in the computation of the optimal transfer orbit. The algorithm requires only minor modifications.

Conclusions

A closed-loop steering logic has been derived for driving the satellite launch vehicle along a three-dimensional trajectory. The final desired orbit is assumed to be circular. The guidance logic makes use of the required velocity concept. The definition of the required velocity is based on the approximation of the future powered trajectory by a two-impulse transfer orbit. This orbit connects the initial elliptical orbit, hypothetically obtained by cutting off the thrust at the current instant, to the desired circular orbit. To simplify the analysis without sacrificing the optimality, the apogee of the transfer trajectory is assumed to lie on the final circular orbit.

The orbital elements are used to describe the initial elliptical orbit, the coasting transfer (elliptical) orbit, and the final circular orbit. Using these, the characteristic velocity function for a two-impulse transfer has been defined. Using the parametric optimization technique, the optimal transfer orbit is determined. In the process, the optimal injection point is also fixed. Once the optimal transfer trajectory is defined, the required velocity and $\boldsymbol{Q}$-matrix are evaluated. The perturbation analysis has been employed to arrive at the optimal solution.

The $\boldsymbol{Q}$-guidance uses the velocity to be gained for generating the steering logic. Instead of computing the velocity to be gained at every time, the VG differential equation is used. Such a guidance works at two levels. The guidance major cycle updates the VG differential equation. The guidance minor cycle, which works within the major cycle, determines the thrust attitude angle by making use of the cross product steering

Appendix

Orbital Parameters for the Initial Orbit i_0, Desired Circular Orbit d

At the initial point I_1, the sensor data determine the position and velocity vectors

$$\boldsymbol{r}^T = (r_1, r_2, r_3), \qquad \boldsymbol{v}_i^T = (v_1, v_2, v_3)$$

from which the elements of the resulting "initial orbit" i_0 can be computed, using the following relations:[19]

$$h^2 = (r_1v_2 - r_2v_1)^2 + (r_2v_3 - r_3v_2)^2 + (r_1v_3 - r_3v_1)^2 \tag{A1}$$

$$\tan\Omega = (r_2v_3 - r_3v_2)/(r_1v_3 - r_3v_1) \tag{A2}$$

$$\cos i = (r_1v_2 - r_2v_1)/h \tag{A3}$$

$$r^2 = (r_1^2 + r_2^2 + r_3^2) = \boldsymbol{r} \cdot \boldsymbol{r}, \quad v_i^2 = (v_1^2 + v_2^2 + v_3^2)$$

$$a = 1/(2/r - v_i^2/\mu)^{1/2} \tag{A4}$$

$$p = a(1 - e^2) = h^2/\mu \tag{A5}$$

$$e^2 = 1 - h^2/\mu a = 1 - p/a \tag{A6}$$

$$\cos\theta = (h^2/\mu r - 1)/e = (p/r - 1)/e \tag{A7}$$

$$\tan(\theta + \omega) = (r_3 \sin i - r_1 \sin\Omega \cos i + r_2 \cos\Omega \sin i)$$
$$\div (r_1 \cos\Omega + r_2 \sin\Omega) \tag{A8}$$

$$\tan(E/2) = ((1 - e)/(1 + e))^{1/2} \tan(\theta/2)$$

The orbital parameters for the desired circular orbit, namely, radius r_d, velocity $v_d = (\mu/r_d)^{1/2}$, inclination i_d, and the argument of the ascending node Ω_d, are specified.

Optimum Range Angle ϕ_d^* Computation

The cost functional associated with the optimization of ϕ_d is given by

$$J = \mu[2p_c/r_d^2 - 2ee_c \sin\delta \sin\theta/(pp_c)^{1/2} - 2(pp_c)^{1/2} \cos\gamma_1/r^2$$
$$- 2(p_c/r_d^3)^{1/2} \cos\gamma_2 + v_i^2 + 2(r_d - r)/rr_d + 1/r_d \tag{A9}$$

The necessary condition for optimality for this unconstrained problem can be expressed as

$$\partial J/\partial\phi_d = 0$$

as ϕ_d is the only variable under control. Using Eq. (A9) for J, one gets the necessary condition for optimality, on simplification,

$$[(2/r_d^2 + ee_c \sin\delta \sin\theta/(pp_c^3)^{1/2} - (p/p_c)^{1/2} \cos\gamma_1/r^2$$
$$- \cos\gamma_2/(p_c r_d^3)^{1/2})\partial p_c/\partial\phi_d - (2e \sin\delta \sin\theta/(pp_c)^{1/2})\partial e_c/\partial\phi_d$$
$$- (2ee_c \cos\delta \sin\theta/(pp_c)^{1/2})\partial\delta/\partial\phi_d + (2(pp_c)^{1/2} \sin\gamma_1/r^2)$$
$$\times \partial\gamma_1/\partial\phi_d + 2(p_c/r_d^3)^{1/2} \sin\gamma_2\, \partial\gamma_2/\partial\phi_d] = 0 \tag{A10}$$

The partial derivatives $\partial p_c/\partial\phi_d$, $\partial e_c/\partial\phi_d$, $\partial\delta/\partial\phi_d$, $\partial\gamma_1/\partial\phi_d$, and $\partial\gamma_2/\partial\phi_d$ required in Eq. (A10) are determined as follows:

On differentiating Eq. (6) with respect to ϕ_d, one gets

$$-\sin\delta\, \partial\delta/\partial\phi_d = -\cos(\beta_1 - \theta)\sin(\beta_2 + \phi_d) - \sin(\beta_1 - \theta)$$
$$\times \cos(\beta_2 + \phi_d)\cos\phi$$

Using Eqs. (8) and (9), respectively, for $1/\sin\delta$ in the first and second terms on the right-hand side of the above equation,

$$\begin{aligned}\partial\delta/\partial\phi_d &= [\sin\gamma_1\cos(\beta_1-\theta)+\sin\gamma_2\cos(\beta_2+\phi_d)\cos\phi]/\sin\phi\\ &=[\sin\gamma_1\cos(\beta_1-\theta)\sin\phi+\cos\gamma_1\cos\phi-\cos\gamma_1\cos\phi)\\ &\quad+(\sin\gamma_2\cos(\beta_2+\phi_d)\cos\phi\sin\phi+\cos\gamma_2\cos^2\phi\\ &\quad-\cos\gamma_2\cos^2\phi)]/\sin^2\phi\end{aligned}$$

After using Eqs. (10) and (11) in the above relation and simplifying,

$$\partial\delta/\partial\phi_d=\cos\gamma_2 \tag{A11}$$

Similarly, from Eqs. (3-5) and (A11), one gets

$$\begin{aligned}\partial p_c/\partial\phi_d &= rr_d e_c\sin\delta\cos\gamma_2/(r_d-r\cos\delta)\\ &=p_c e_c\cos\gamma_2[(1+\cos\delta)/(1-\cos\delta)]^{1/2}\end{aligned} \tag{A12}$$

$$\partial e_c/\partial\phi_d=-re_c\sin\delta\cos\gamma_2/(r_d-r\cos\delta) \tag{A13}$$

from Eqs. (8, 10, A11, 7)

$$\partial\gamma_1/\partial\phi_d=\sin\gamma_2/\sin\delta \tag{A14}$$

and from Eqs. (9) and (A11)

$$\partial\gamma_2/\partial\phi_d=-\sin\gamma_2\cot\delta \tag{A15}$$

Using Eqs. (A11-A15) in Eq. (A10), the condition for optimality is obtained as

$$\begin{aligned}&[(2/r_d^2+ee_c\sin\delta\sin\theta/(pp_c^3)^{1/2}-(p/p_c)^{1/2}\cos\gamma_1/r^2\\ &-\cos\gamma_2/(p_c r_d^3)^{1/2})(rr_d e_c\sin\delta\cos\gamma_2/(r_d-r\cos\delta))\\ &+(2e\sin\delta\sin\theta/(pp_c)^{1/2})(re_c\sin\delta\cos\gamma_2/(r_d-r\cos\delta))\\ &-2ee_c\cos\delta\cos\gamma_2\sin\theta/(pp_c)^{1/2}+2(pp_c)^{1/2}\sin\gamma_1\sin\gamma_2/r^2\sin\delta\\ &-2(p_c/r_d^3)^{1/2}\sin\gamma_2\cot\delta]=0\end{aligned} \tag{A16}$$

For brevity, let

$$\begin{aligned}&x_1=\cos\delta=\cos(\beta_1-\theta)\cos(\beta_2+\phi_d)-\sin(\beta_1-\theta)\sin(\beta_2+\phi_d)\cos\phi\\ &x_2=\sin\delta,\quad x_3=\sin\delta\cos\gamma_2,\quad x_4=\sin\delta\cos\gamma_1\\ &x_5=r_d-r\cos\delta=r_d-rx_1,\quad x_6=(1-x_1)^{1/2},\quad x_7=(1+x_1)^{1/2}\\ &x_8=r/r_d\end{aligned} \tag{A17}$$

Using Eqs. (10) and (11), it can be shown that

$$x_3=\cos(\beta_1-\theta)\sin(\beta_2+\phi_d)+\sin(\beta_1-\theta)\cos(\beta_2+\phi_d)\cos\phi$$

$$x_4=\cos(\beta_1-\theta)\sin(\beta_2+\phi_d)\cos\phi+\sin(\beta_1-\theta)\cos(\beta_2+\phi_d)$$

Thus, it can be shown that the variables x_i and $i=1,\ldots,8$ can be expressed explicitly in terms of ϕ_d and known variables. Using these relations, the optimality conditions are

$$\begin{aligned}&[2(1-x_8)x_8^{1/2}x_3/x_6x_5^{3/2}+er_d(1-x_8)^2x_2x_3\sin\theta/(1-x_1)^2x_5\\ &-(r_d-r)x_3x_4p^{1/2}/(1-x_1)x_2x_5-(r_d-r)x_3^2/r_d^{3/2}(1-x_1)x_5x_2\\ &+2e(r_d-r)x_2x_3\sin\theta/p^{1/2}(1-x_1)x_5\\ &-2e(r_d-r)x_1x_3\sin\theta/r_d r(1-x_1)x_2p^{1/2}\\ &+2p^{1/2}\sin(\beta_2+\phi_d)\sin(\beta_1-\theta)\sin^2\phi/r^2x_2^3\\ &-2\sin^2(\beta_1-\theta)\sin^2\phi/r_d^{3/2}x_1x_2]=0\end{aligned} \tag{A18}$$

Derivation of Q-Matrix

For the coasting trajectory, the principles of conservation of angular momentum and energy hold good, i.e.,

$$\begin{aligned}\boldsymbol{h}&=\boldsymbol{r}\times\boldsymbol{v}_r=\text{constant vector}\\ E&=v_r^2/2-\mu/r=\text{constant}\end{aligned} \tag{A19}$$

where $\boldsymbol{r}=(r_1,r_2,r_3)$, $\boldsymbol{v}_r=(v_{r1},v_{r2},v_{r3})$, and

$$r^2=(r_1^2+r_2^2+r_3^2),\quad v_r^2=(v_{r1}^2+v_{r2}^2+v_{r3}^2)$$

The components of $\boldsymbol{h}$ can be written as

$$h_1=r_2v_{r3}-r_3v_{r2} \tag{A20}$$

$$h_2=r_3v_{r1}-r_1v_{r3} \tag{A21}$$

$$h_3=r_1v_{r2}-r_2v_{r1} \tag{A22}$$

On multiplying Eqs. (A6-A8) by v_{r1}, v_{r2}, v_{r3} respectively, and adding, one gets

$$h_1v_{r1}+h_2v_{r2}+h_3v_{r3}=0$$

which, on differentiating with respect to $\boldsymbol{r}$, yields

$$h_1\partial v_{r1}/\partial\boldsymbol{r}+h_2\partial v_{r2}/\partial\boldsymbol{r}+h_3\partial v_{r3}/\partial\boldsymbol{r}=0 \tag{A23}$$

Similarly, differentiating E with respect to $\boldsymbol{r}$ gives

$$v_{r1}\partial v_{r1}/\partial\boldsymbol{r}+v_{r2}\partial v_{r2}/\partial\boldsymbol{r}+v_{r3}\partial v_{r3}/\partial\boldsymbol{r}=\mu\boldsymbol{r}/r^3 \tag{A24}$$

Further, differentiating Eqs. (A20), (A21), and (A22) with respect to r_1, r_2, and r_3, respectively, yields

$$r_2\partial v_{r3}/\partial r_1-r_3\partial v_{r2}/\partial r_1=0 \tag{A25}$$

$$r_3\partial v_{r1}/\partial r_2-r_1\partial v_{r3}/\partial r_2=0 \tag{A26}$$

$$r_1\partial v_{r2}/\partial r_3-r_2\partial v_{r1}/\partial r_3=0 \tag{A27}$$

The elements of the $\boldsymbol{Q}$-matrix are defined below:

$$\boldsymbol{Q}=\begin{bmatrix}\partial v_{r1}/\partial r_1 & \partial v_{r1}/\partial r_2 & \partial v_{r1}/\partial r_3\\ \partial v_{r2}/\partial r_1 & \partial v_{r2}/\partial r_2 & \partial v_{r2}/\partial r_3\\ \partial v_{r3}/\partial r_1 & \partial v_{r3}/\partial r_2 & \partial v_{r3}/\partial r_3\end{bmatrix}$$

Using Eqs. (A23-A27), and after considerable simplification, the $\boldsymbol{Q}$-matrix can be reduced to

$$\boldsymbol{Q}=\mu/r^3(r_1v_{r1}+r_2v_{r2}+r_3v_{r3})\begin{bmatrix}r_1^2 & r_1r_2 & r_1r_3\\ r_2r_1 & r_2^2 & r_2r_3\\ r_3r_1 & r_3r_2 & r_3^2\end{bmatrix} \tag{A28}$$

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